VaR and CVaR: A Non-normal Regime Switching Framework

Hong Miao
Haskayne School of Business

October 11, 2007

Outline

Outline

1. *Introduction*.
2. *Methodology*.

Outline

1. *Introduction*.
2. *Methodology*.
3. *Techniques*.

Outline

1. *Introduction*.
2. *Methodology*.
3. *Techniques*.
4. *Main Results*.

Purpose

- Develop a method for computing VaR and CVaR in a regime switching framework.

Purpose

- Develop a method for computing VaR and CVaR in a regime switching framework.
- Capture the fat-tailed phenomenon by the Student's T distributions.

Purpose

- Develop a method for computing VaR and CVaR in a regime switching framework.
- Capture the fat-tailed phenomenon by the Student's T distributions.
- Capture the volatility clustering phenomenon by the "regimes".

Definitions (1)

Definition

Risk: Roughly speaking, risk means a chance of injury or loss associated with a given action.

Definitions (2)

Definition

VaR is a statistical estimation of a portfolio loss with the property that, with a small probability α, the owner of the portfolio stands to incur that loss or more over a given (typically short) holding period.

Definitions (3)

Definition

CVaR equals the conditional expectation of X, given that $X > VaR_{\alpha}(X)$, i.e.

$$CVaR_{\alpha} = E\left[X | X > VaR_{\alpha}(X)\right].$$

Literature (1)

- Markov Switching models have proved to be useful for modeling a range of economic time series.

Literature (1)

- Markov Switching models have proved to be useful for modeling a range of economic time series.
- Business cycle (Hamilton 1989, Goodwin 1993; Durland and McCurdy 1994; Layton and Smith 2000)

Literature (1)

- Markov Switching models have proved to be useful for modeling a range of economic time series.
- Business cycle (Hamilton 1989, Goodwin 1993; Durland and McCurdy 1994; Layton and Smith 2000)
- Stock market (Turner, Startz and Nelson 1989; Hamilton and Susmel 1994; Schaller and van Norden 1997)

Literature (1)

- Markov Switching models have proved to be useful for modeling a range of economic time series.
- Business cycle (Hamilton 1989, Goodwin 1993; Durland and McCurdy 1994; Layton and Smith 2000)
- Stock market (Turner, Startz and Nelson 1989; Hamilton and Susmel 1994; Schaller and van Norden 1997)
- Exchange Rates (Engle and Hamilton 1990)

Literature (1)

- Markov Switching models have proved to be useful for modeling a range of economic time series.
- Business cycle (Hamilton 1989, Goodwin 1993; Durland and McCurdy 1994; Layton and Smith 2000)
- Stock market (Turner, Startz and Nelson 1989; Hamilton and Susmel 1994; Schaller and van Norden 1997)
- Exchange Rates (Engle and Hamilton 1990)
- Short-term interest rates (Cai 1994, Gray 1996)

Literature (2)

- Guidolin and Timmermann (2003) provide a four state multivariate regime switching model for the joint distribution of monthly stock and bond returns. They found that the regime switching model is accurate in out-of-sample VaR predictions.

Literature (2)

- Guidolin and Timmermann (2003) provide a four state multivariate regime switching model for the joint distribution of monthly stock and bond returns. They found that the regime switching model is accurate in out-of-sample VaR predictions.
- Kawata and Kijima (2005) introduce adjustment factors for state probabilities in regime switching models for estimate portfolio VaR, and argue that their models perform well.

Regime Switching

- We consider a two state regime switching model.

Regime Switching

- We consider a two state regime switching model.
- We represent such "states of the world" by a Markov chain X with a transition matrix A.

Regime Switching

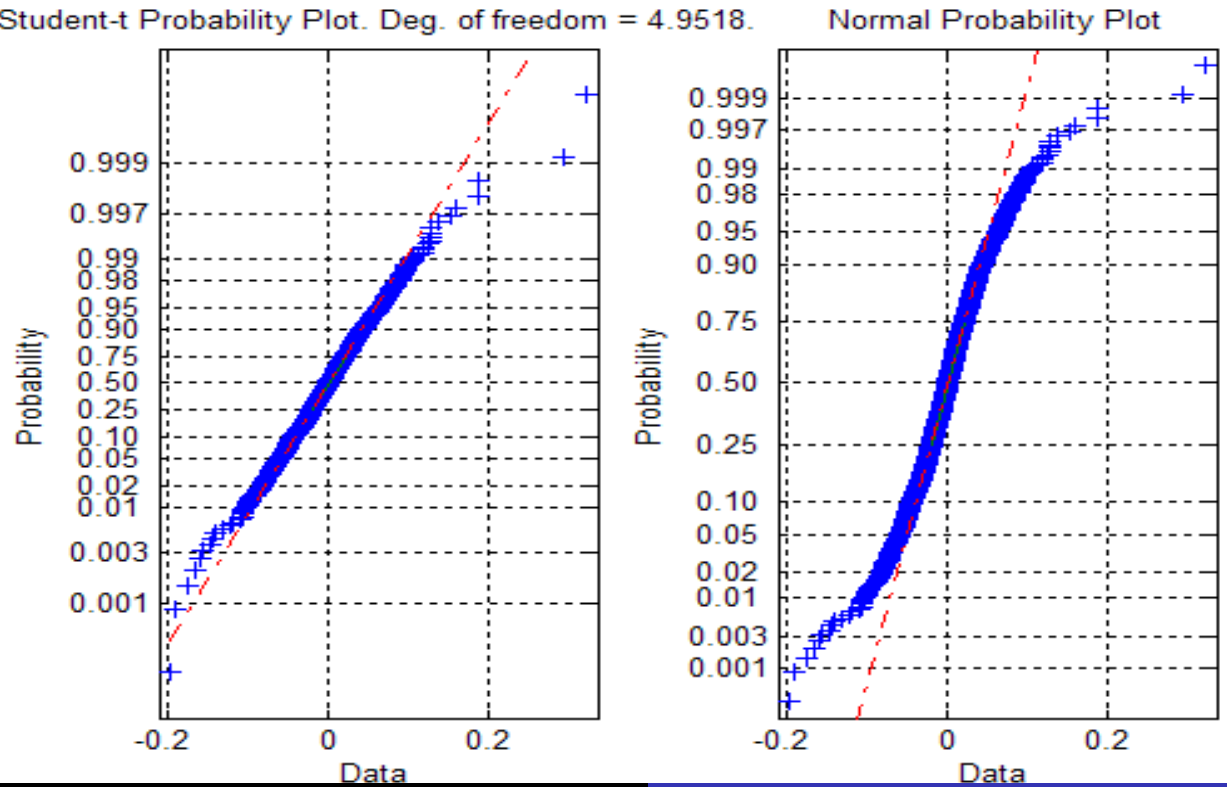
- We consider a two state regime switching model.
- We represent such "states of the world" by a Markov chain X with a transition matrix A.
- We suppose the returns $Y_t \, (0 \leq t \leq T)$ of an individual financial asset, S, follow a switching t-distribution.

Why Student-t Distribution (1)

- Lin and Shen found that using the Student-t distribution for estimating VaR can improve the VaR estimation and offer accurate VaR estimates in their The Journal of Risk Finance (2006) paper.

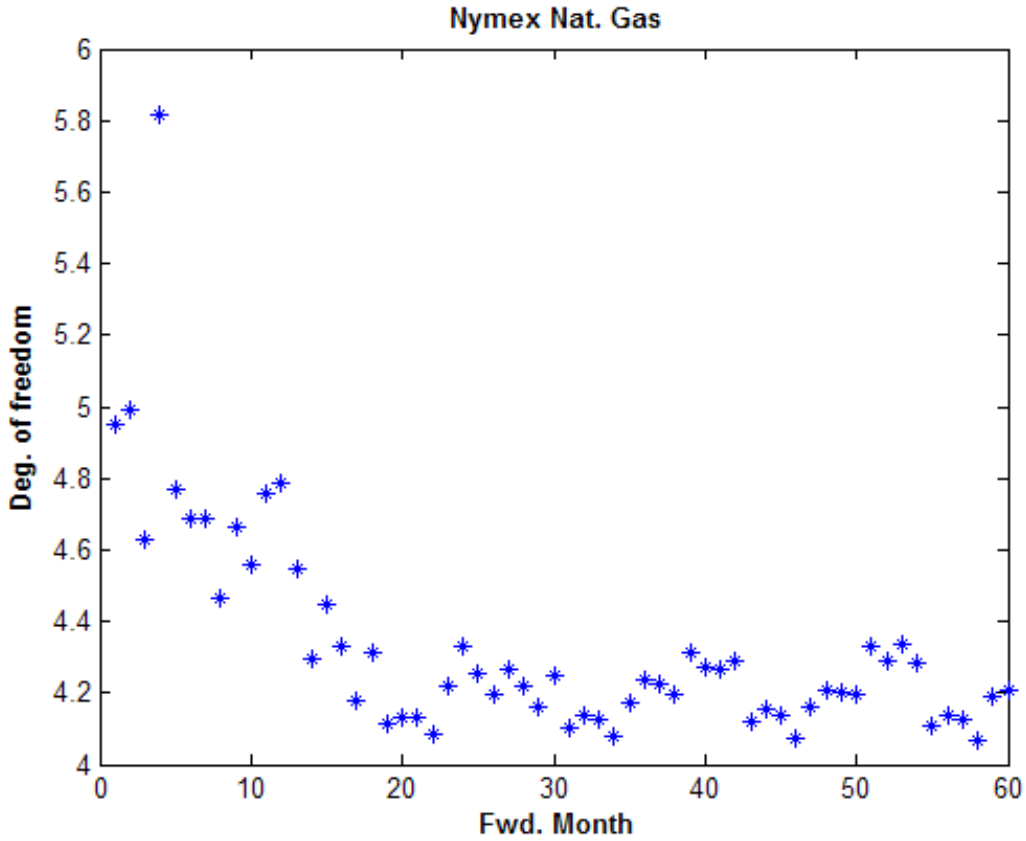
Why Student-t Distribution (2)

- The following picture compares Student-t and Normal distributions applied to the prompt month Hub Nat. gas Nymex Futures contract.



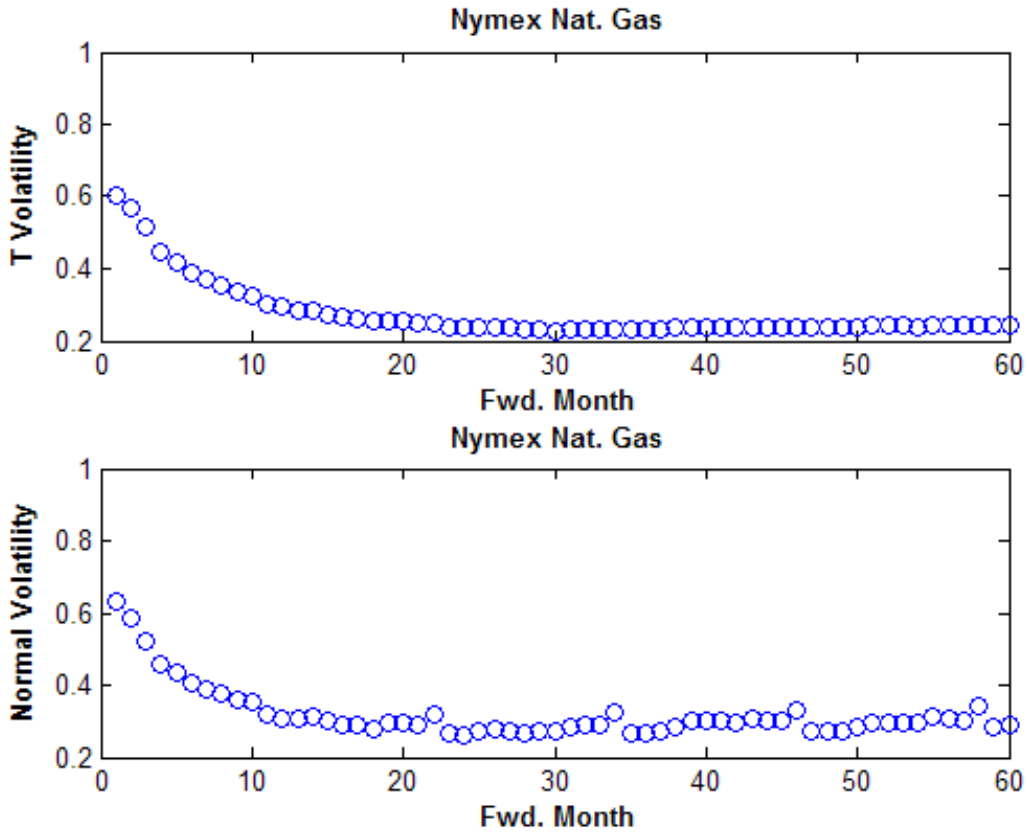
Why Student-t Distribution (3)

- The following graph shows the Degree of Freedom estimation for the Hub Nat. gas Nymex Futures contract.



Why Student-t Distribution (4)

- The following graph shows that Student-t estimates are more stable.



Change of Measure

- In practice, we only observe the return process $Y_1, Y_2, ..., Y_t$,

Change of Measure

- In practice, we only observe the return process $Y_1, Y_2, ..., Y_t$,
- We want to estimate the transition matrix A, and the means and variances in the model based on the available information.

Change of Measure

- In practice, we only observe the return process $Y_1, Y_2, ..., Y_t$,
- We want to estimate the transition matrix A, and the means and variances in the model based on the available information.
- We initially work with a "reference" probability $\bar{P}$ under which the Y_t are a sequence of i.i.d. random variables independent of X and each of which is $T(1, 0, 1)$.

Change of Measure

- In practice, we only observe the return process $Y_1, Y_2, ..., Y_t$,
- We want to estimate the transition matrix A, and the means and variances in the model based on the available information.
- We initially work with a "reference" probability $\bar{P}$ under which the Y_t are a sequence of i.i.d. random variables independent of X and each of which is $T(1, 0, 1)$.
- We than change measure back to the real probability space.

EM Algorithm

The basic idea of the EM algorithm is:

1. We start with appropriate initial values $\hat{\theta}_0$ for

$$\theta := \{p_{ji}, \mu_i, \nu_i, \theta_i, \ 1 \leq i, j \leq 2\}$$

which satisfy constraints for the parameters.

EM Algorithm

The basic idea of the EM algorithm is:

1. We start with appropriate initial values $\hat{\theta}_0$ for

$$\theta := \{p_{ji}, \mu_i, \nu_i, \theta_i, \ 1 \leq i, j \leq 2\}$$

   which satisfy constraints for the parameters.
2. After some observations of Y, we compute the new estimates.

EM Algorithm

The basic idea of the EM algorithm is:

1. We start with appropriate initial values $\hat{\theta}_0$ for

$$\theta := \{p_{ji}, \mu_i, \nu_i, \theta_i, \ 1 \leq i, j \leq 2\}$$

   which satisfy constraints for the parameters.
2. After some observations of Y, we compute the new estimates.
3. Using these values, we re-estimate the parameters iteratively until some stopping criterion is satisfied.

EM Algorithm

The basic idea of the EM algorithm is:

1. We start with appropriate initial values $\hat{\theta}_0$ for

$$\theta := \{p_{ji}, \mu_i, \nu_i, \theta_i, \ 1 \leq i, j \leq 2\}$$

   which satisfy constraints for the parameters.
2. After some observations of Y, we compute the new estimates.
3. Using these values, we re-estimate the parameters iteratively until some stopping criterion is satisfied.
4. After more observations we repeat the process again.

- Derive closed-form formulas to filter out all parameters of the model and.

- Derive closed-form formulas to filter out all parameters of the model and.
- Closed-form VaR and CVaR for a single asset.

- Derive closed-form formulas to filter out all parameters of the model and.
- Closed-form VaR and CVaR for a single asset.
- Closed-form VaR and CVaR for a Portfolio.

- Just a toy model.

- Just a toy model.
- It might be interesting to check its applications by some real world data.

- Just a toy model.
- It might be interesting to check its applications by some real world data.
- It might be interesting for industry people.

Thanks a lot for your time and attentions.

Suggestions and comments are appreciated.