Quantitative Strategies

Inflation Swaps and Corporate Pension Plans

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Executive Summary

Liability driven investments require a new set of tools for asset managers and pension fund trustees. The complexity of hedging calculations is often increased due to indexation policies that transform pension obligations into real liabilities. Inflation swaps will reduce this complexity. They can simply be overlaid on an existing manager roaster without interfering with the current mandate structure or interest rate exposure. While this makes inflation hedging highly practical (easy to implement) it is not clear that it is equally desirable for corporate pension funds. In fact corporate finance theory dictates us that there is only limited scope for it.

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1. The Mechanics of Inflation Swaps

The management of pension fund risk is an integral part of shareholder value maximizing corporate risk management.[1] As such, pension liabilities are subject to interest rate risk, credit risk and inflation risk. While the management of the former two is already well described in the existing literature[2] the material on inflation derivatives is somewhat sparse[3]. This article attempts at providing insights into the use of inflation swaps[4] when managing the risk of corporate pension liabilities.

With the advent of inflation derivatives plan sponsors can take out (or build) risks like a surgeon. For example a UK pension fund can invest into a portfolio of nominal UK government bonds to get UK interest rate exposure, invest into inflation swaps to get inflation exposure for indexed liabilities and on top of this sell credit default swaps to match the plans credit risk with that of the sponsor. Without derivatives the same pension fund could have invested into corporate bonds from the same rating category. However these bonds would very unlikely offer inflation protection nor will the available duration be long enough to close the duration gap.

The inflation swap market is an OTC market that derives its attractiveness from its flexibility. Swaps are customisable with respect to the specification of the relevant inflation index as well as maturity structure.[5] Currently indexation is capped at 5% per annum. This limited RPI participation could be easily dealt with swap arrangements. While inflation swaps come in many forms (zero coupon swaps, revenue inflation swap, year on year inflation swaps, etc.) we will focus on zero coupon inflation swaps. They are the most typical contract form and hence offer most liquidity. Market participants that are naturally long inflation, i.e. those which receive inflation and are hence willing to sell it in order to reduce their cash flow uncertainty are

- **Regulated industries:** **RPI+X schemes can be found in water, gas and other utilities and effectively tie profits to the retail price index (RPI).**
- **House-owners:** **Inflation indexed rents link rental income positively to inflation.**
- **Retailers:** **Selling a broad basket of goods builds an indirect inflation exposure as long as margins remain stable.**
- **Government agencies:** **Collecting revenue related to VAT and wage related income tax.**

Participants that are naturally short inflation and want to receive it are to a large degree investors with real liabilities like pension funds.[6]

How do the mechanics of inflation swaps work? At maturity t_n the fixed rate payer (for example UK pension fund) pays the fixed amount

$$N\left[\left(1+K_{t_n}\right)^{t_n}-1\right], \qquad \textbf{1)}$$

where N denotes the contract nominal and K_{t_n} reflects the fixed contractually agreed swap rate. In exchange the party receives a floating amount of

$$N\left(\frac{I_{t_n}}{I_{t_0}}-1\right) \qquad \textbf{2)}$$

from a UK utility that aims to unload its inflation exposure. Note that I_{t_n} and I_{t_0} stand for the CPI (consumer price inflation, or indeed any other inflation measure) levels at dates t_n and t_0 respectively. Another way to look at (2) is to describe $I_{t_n}/I_{t_0}=\left(1+\bar{\pi}\left(0,t_n\right)\right)^{t_n}$ as the *cumulated realized inflation* at maturity, where $\bar{\pi}\left(0,t_n\right)$ denotes its geometric average. Note that (1) and (2) also imply that no principal is being exchanged at maturity and no cash flows occur between $t_0,\ldots,t_n$. At the time when both parties enter the swap it has a market value of zero, i.e. floating and fixed side are set to be equal in value. The difference in price between a real bond $b(0,t_n)$ and a nominal bond $d(0,t_n)$ must reflect the markets expectations of cumulative inflation which as we know from above defines the value of the floating leg. In turn the fixed leg consists of the known fixed (nominal) payment at t_n discounted by the appropriate discount rate at $t_0=0$. We therefore get

[1] See Scherer (2006) for a simple model of how hedging asset liability risk can release risk capital that can otherwise be used for investments into projects with positive net present value.
[2] See Duffie/Singleton (2003) on the principles of managing credit risk or Tuckman (1996) on interest rate risk. Scherer (2005) applies these principles to liability hedging.
[3] See Benaben (2005) on a non-technical review of inflation linked derivatives
[4] Inflation swaps are contractual agreements, where one party receives realized inflation, while the other party pays a fixed rate.
[5] Liquidity still is an issue in both the inflation swap as well as the market for index linked gilt. Few corporate deals are known and the market is anything else than a two way market with currently high bid ask spreads.
[6] In fact any investor that wants to protect the real value of its investments needs to find exposure to inflation.

$$\underbrace{N \cdot \left[b(0,t_n) - d(0,t_n) \right]}_{\text{floating leg}} = \underbrace{d(0,t_n) \cdot N \cdot \left[\left(1+K_{t_n}\right)^{t_n} - 1 \right]}_{\text{fixed leg}} \qquad \textbf{3)}$$

with the definitions for real and nominal bonds given below

$$b(0,t_n) = \left(\frac{1}{1+r(0,t_n)}\right)^{t_n}, \quad d(0,t_n) = \left(\frac{1}{1+s(0,t_n)}\right)^{t_n} \qquad \textbf{4)}$$

We write $r(0,t_n), s(0,t_n)$ to denote real and nominal market yields over period t_n. Equation (3) is derived simply by arbitrage arguments and will hold under any term structure or inflation model. Simplifying (3) we arrive at what we finally want: the relationship between real bonds, nominal bonds and implied inflation:

$$b(0,t_n) = d(0,t_n) \cdot \left(1+K_{t_n}\right)^{t_n} \qquad \textbf{5)}$$

How can we fill the mathematical identity (5) with economic life? The difficulty is that arbitrage only ensures relative pricing. In practice we therefore need to pick two elements in (5) which we believe are fairly valued in order to price the third component. In principle we can derive $b(0,t_n)$ from inflation linked coupon bonds as we can derive $d(0,t_n)$ from nominal coupon bonds to essentially arrive at K_{t_n}. However this is a difficult exercise that requires considerable time and effort.[7] For a pension fund this is best left to investment banks. One would hope that competition will ensure that the observed K_{t_n} do not leave excess profits for market makers in inflation swaps. Quotations for inflation swaps in the OTC market are provided in Table 1. The market for zero coupon inflation swaps quotes fixed payments as a function of swap maturity K_{t_n}, which we can also interpret as the *market implied (geometric average) inflation.* Note that this is different from an arbitrary measure of expected inflation which would differ between market participants.[8]

t_n	$K_{t_n}^{bid}$	$K_{t_n}^{ask}$	$K_{t_n}^{mid}$
1	2.30%	2.60%	2.45%
2	2.33%	2.57%	2.45%
3	2.36%	2.60%	2.48%
4	2.42%	2.66%	2.54%
5	2.48%	2.72%	2.60%
6	2.54%	2.78%	2.66%
7	2.60%	2.84%	2.72%
8	2.64%	2.88%	2.76%
9	2.68%	2.92%	2.80%
10	2.71%	2.95%	2.83%
12	2.76%	3.00%	2.88%
15	2.83%	3.07%	2.95%
20	2.94%	3.18%	3.06%
25	2.99%	3.23%	3.11%
30	3.01%	3.25%	3.13%

Table 1. Market quotes on zero coupon inflation swaps. I_{t_0}=193.4. Source: ICAP on Reuters, March 14th, 2006

For example the implied UK inflation (measured as RPI) for a period of 20 years is 2.94% per annum, which equals 345.25. Compared with market implied inflation for the French CPI for the same date, this is considerably higher (about 75 basis points per annum).[9] This could be a reflection of the economic outlook as well as the weaker inflation fighting credibility of the Bank of England.

2. Swap Sensitivities

In risk management applications we require the sensitivities of inflation swaps to changes in both real rates as well as inflation. In order to set up the initial hedge ratios for a duration hedge we need to look at the swap at the time both parties enter it. From the above analysis we know the initial value of an inflation swap is zero.

$$swap = b(0,t_n) - d(0,t_n) \cdot \left(1+K_{t_n}\right)^{t_n} = 0 \qquad \textbf{6)}$$

We start with calculating the sensitivity of (6) with respect to changes in the markets implied inflation expectations. Let denote the implied inflation expectation from zero inflation swaps. When both parties enter an inflation swap agreement the fixed swap rate and the markets expectations by definition coincide, i.e. $K_{t_n} = \pi(0,tn)$. Taking derivatives we get

$$\left.\frac{dswap}{dr}\right|_{K_{t_i}=\pi(0,t_n)} = \frac{db(0,t_n)}{dr} - \frac{dd(0,t_n)}{dr}\left(1+K_{t_n}\right)^{t_n}$$
$$= -t_n\left(1+r(0,t_n)\right)^{-t_n-1} + t_n\left(1+\pi(0,t_n)\right)^{-t_n}\left(1+r(0,t_n)\right)^{-t_n-1}\left(1+K_{t_n}\right)^{t_n} = 0 \qquad \textbf{7)}$$

[7] See Evans (1998) and Anderson/Sleath (2001) for a technical summary of alternative methods.

[8] While we can not use the path of expected inflation (for example out of a macroeconomic scenario generation model) to price inflation swaps, it will be individual inflation expectations that let investors decide whether to invest in these instruments or not.

[9] Data not shown in text.

$$\left.\frac{dswap}{d\pi}\right|_{K_{t_i}=\pi(0,t_n)} = \frac{db(0,t_n)}{d\pi} - \frac{dd(0,t_n)}{d\pi}\left(1+K_{t_n}\right)^{t_n}$$
$$= 0 + t_n\left(1+\pi(0,t_n)\right)^{-t_n-1}\left(1+r(0,t_n)\right)^{-t_n}\left(1+K_{t_n}\right)^{t_n}$$
$$= t_n\frac{b(0,t_n)}{1+\pi(0,t_n)} > 0 \qquad \mathbf{8)}$$

The real duration of an inflation swap (at initiation) is zero while its inflation duration is positive.[10] In other words the swap will gain in value, if the markets (implied) inflation expectation rises. Inflation swaps offer therefore a convenient way to change the inflation duration to any desired level, while leaving real duration unaffected. This makes them very attractive for overlay management. Pension plans can now engineer any desired exposure as summarized in Table 2. A roaster of market neutral alpha generators[11] will typically generate the return of short term money market instruments (cash) plus out-performance (alpha). However, realizing that cash is one of the most risky assets in an asset liability framework, because of its zero duration the pension fund needs to overlay the multi-manager allocation with an interest rate swap. However the return on this structure might not be enough to meet the required discount rate as it might contain additional company specific credit risk. Therefore the pension fund moves down another row to add credit risk. On top of this inflation swaps can be used to overlay the existing structure with an inflation hedge. Again the beauty of using inflation swaps (as well as credit default or interest rate swaps) is that they can be overlaid on any existing fixed income or equity portfolio without interfering with either manager structure or targeted interest rate exposure.

Investment	Purpose	Pay	Receive	Total
Market neutral manager structure	Alpha generation	—	Cash + α	Cash + α?
n-year interest rate swap	Create duration	Cash	n-year rate	n-year rate + α
Credit default swap	Create credit exposure	Conditional on default	Credit risk premium	n-year swap rate + α + credit risk premium
Inflation swap	Create inflation	Expected exposure	Realized Inflation	n-year Inflation real rate + α + credit risk premium

Table 2. Overlay structure. Inflation swaps can be used to match inflation sensitivities within a standard building block approach to engineer desired risk exposures.

It however must be stressed that strictly speaking the notion of zero sensitivity to real rates might be somewhat misleading as (7) is only valid at the trade outset. While the fixed rate will remain unchanged over the life of the swap the markets implied expectations will change and inflation will accrue. Building on our intuition from above, we can now answer a set of more general questions:

- What is the value of a zero coupon inflation swap at any time t_i, given that we have entered it at time 0 with maturity t_n?
- How does this affect its sensitivity with respect to real rate changes?

Let us define the *realized* average inflation (geometric) over the time interval between 0 and t_i as $\bar{\pi}(0,t_i)$ so that the accrued inflation becomes $\left(1+\bar{\pi}(0,t_i)\right)^{t_i}$. Equation (6) then becomes

$$swap(t_i,t_n) = \left(1+\pi(0,t_i)\right)^{t_i} b(t_i,t_n) - d(t_i,t_n)\cdot\left(1+K_{t_n}\right)^{t_n} \qquad \mathbf{9)}$$

Note that $b(t_i,t_n)$ is the price of a real zero bond with maturity (t_i,t_n) and real interest rate $r(t_i,t_n)$, i.e. the real interest rate at time t_i for maturity t_n. If the inflation swap approaches maturity $t_i = t_n$ equation (9) will reduce to $\left(1+\bar{\pi}(0,t_n)\right)^{t_n} - \left(1+K_{t_n}\right)^{t_r}$ as $b(t_n,t_n) = d(t_n,t_n) = 1$. This equals the definition of the final payout of an inflation swap, i.e. the exchange of realized inflation against a fixed payment. If we now calculate the inflation sensitivity we get a slightly different result.

$$\frac{dswap(t_i,t_n)}{dr} = \left(1+\bar{\pi}(0,t_i)\right)^{t_i}\frac{db(t_i,t_n)}{dr} - \frac{dd(t_i,t_n)}{dr}\cdot\left(1+K_{t_n}\right)^{t_n}$$
$$= -\left(1+\bar{\pi}(0,t_i)\right)^{t_i}(t_n-t_i)\frac{b(t_i,t_n)}{1+r(t_i,t_n)} + (t_n-t_i)\frac{d(t_i,t_n)}{1+r(t_i,t_n)}\cdot\left(1+K_{t_n}\right)^{t_n} \qquad \mathbf{10)}$$
$$= -(t_n-t_i)\frac{swap(t_i,t_n)}{1+r(t_i,t_n)}$$

While (10) can become either positive or negative, depending on the present value of the swap agreement, it is most likely to be small as swap value and maturity effect move into different directions. Either the swap value is small, e.g. at contract initiation when maturity is long or the maturity is short, e.g. at maturity when the swap value is potentially large.

[10] Note that the term duration has been borrowed to describe the sensitivity of swap rate changes to changes in inflation and real rate.

[11] The term alpha describes risk adjusted out-performance against a sponsor specified benchmark. Its equivalent in the language of a corporate treasurer is positive EVA (economic value added) or positive NPV (net present value).

3. Market Consistent Valuation of Pension Liabilities

Suppose we are given quotes from the inflation swap market as in Table 1. The actuarial valuation of pension liabilities needs a projected path of future inflation to arrive at projected nominal cash flows that in turn can be discounted with nominal rates. There are many ways to arrive at these inflation rates. They range from consensus forecasts to autoregressive modeling or the estimation of complex multi-equation models.[12] All of these approaches share the same flaw. They build inflation expectations that are most likely to be inconsistent with the pricing of inflation derivative leading to market inconsistent valuations. The wedge between market and actuarial value will make effective hedging of inflation risks impossible. Position to be hedged and hedging instrument will be driven by different factors. This section will therefore provide a toolkit on how to derive the market level of CPI as well as building a monthly CPI curve with seasonality adjustment. First we look at the historical data for monthly changes in UK inflation given in Figure 1, which suggests that the mean inflation is substantially different in each month.

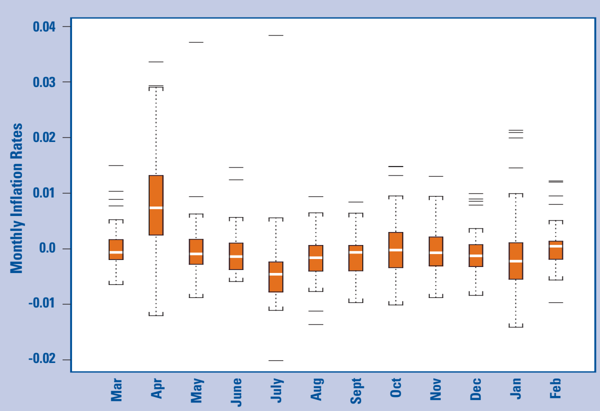

Figure 1. Inflation seasonality. Source: monthly UK RPI data from Datastream (UKCONPRCF) ranging from 1956:1 to 2006:1

Dummy	Value	Std. Error	t value	p value
March	—	—	—	—
April	0.90%	0.0008	11.16	0.00
May	-0.01%	0.0008	-0.09	0.93
June	-0.07%	0.0008	-0.91	0.36
July	-0.40%	0.0008	-5.02	0.00
August	-0.16%	0.0008	-2.01	0.05
September	-0.12%	0.0008	-1.53	0.13
October	0.05%	0.0008	0.58	0.56
November	-0.03%	0.0008	-0.39	0.70
December	-0.08%	0.0008	-1.00	0.32
January	-0.10%	0.0008	-1.26	0.21
February	0.02%	0.0008	0.20	0.84

Table 3. Results from seasonal dummy regression (11). All coefficient estimates are similar in size to the variation shown in Figure 1. They also show the same standard error as they are essentially all vectors of ones and zeros. The R^2 of this regression is 22%.

Next we model seasonality with an additive dummy variable approach, regressing monthly demeaned inflation, π_t, against eleven seasonal dummies, that take on a value of one in the respective month and zero in all other month in order to capture varying mean inflation.

$$\pi_t = D_{Jan}\alpha_{Jan} + D_{Feb}\alpha_{Feb} + D_{Apr}\alpha_{Apr} \ldots + D_{Dec}\alpha_{Dec} + \varepsilon_t \quad \textbf{11)}$$

Note that we left out a seasonal Dummy for March. This is equivalent to enforce zero seasonality in March, or more precisely all other regression coefficients catch seasonality relative to March inflation as all March seasonality is already built into current swap quotes. The results from this regression can be found in Table 3.

Figure 2. Market implied RPI levels. The curve is built using linear inflation growth adjusted for seasonal variation according to (11).

[12]See Wilkie (1986) as the most representative of actuarial dynamic financial analysis.

We can now model market implied levels of RPI for every single month into the future. Figure 2 summarizes the results for the next two years from March 2006 forward. As an example we go through the calculation of the RPI level in August 2007. From Table 1 we know that the 1 year implied inflation (for March 2007) is 2.45%. This leads to an RPI level of 198.14. The same calculation for March 2008 leads to an implied level of $(1+2.45\%)^2 = 202.99$. Linear interpolation between both numbers provides us with the seasonally unadjusted RPI level of 200.16. Given that the inflation in August is on average 0.16% smaller we arrive at the new seasonally adjusted figure of $200.16\,(1-0.16\%) = 199.84$.

Hence, the correctly projected nominal value of an inflation indexed payment in August 2007 is given by

$$C_{March\ 2006} \frac{RPI_{August\ 2007}}{193.4}$$

where $C_{March2006}$ is the March 2006 real value of a 100% indexed payment. This number can in turn be discounted using nominal yields. As an application of this we note that the projected benefit obligation (PBO) is the present value of (currently earned) benefit payments based on projected future salaries. As inflation literally leads to inflated future salaries, pension cash flows increase. This introduces a real element into pension liabilities that can not be addressed with nominal bonds alone.[13]

4. Macro Hedging: Combining Inflation and Interest Duration

Suppose a plan sponsor already used duration to hedge interest rate risk. For example he bought a portfolio of BBB corporate bonds with 5.3 years duration. On top of that he bought interest rate swaps to increase the total duration to 12.7 years which is exactly the interest rate duration of pension liabilities.[14] After having implemented the desired duration hedge we can separately calculate the targeted inflation hedge. How much zero inflation swaps have to be purchased in order to hedge inflation risks? We start with the assumption that there is no basis risk, i.e. the inflation received in the inflation swap is the same inflation that affects both (nominal) bonds as well as pension liabilities. The inflation risk (π) of a given pension fund can then be expressed as

$$\begin{aligned}\Pi &= Var\left(\left(D_\pi^l - D_\pi^n - \phi\, D_\pi^{swap}\right)\Delta\pi\right) \\ &= \left(D_\pi^l - D_\pi^n - \phi\, D_\pi^{swap}\right)^2 Var\left(\Delta\pi\right)\end{aligned} \qquad \textbf{12)}$$

where D_π^x denotes the inflation duration for $x \in \{l,n,swap\}$, i.e. liabilities, nominal bonds and inflation swaps.[15] The hedge ratio ϕ can be initially set independently from the underlying bond portfolio, as inflation swaps do not carry real rate duration. It is simply the value of ϕ that sets the squared term in (12) to zero.

$$\phi = \frac{D_\pi^l - D_\pi^n}{D_\pi^{swap}} \qquad \textbf{13)}$$

Suppose the inflation duration of liabilities is -5. In other words: A 1% increase in inflation would ceteris paribus (i.e. keeping real rates constant) lead to a 5% decrease in liabilities. The value of liabilities would remain constant only if liabilities were perfectly indexed. Assume further the inflation duration of nominal bonds is -10, while the inflation duration of an inflation swap is assumed to be 10. The swap value rises if inflation increases. Substituting these values into (13) yields a 50% hedge ratio. The inflation swap hedges whatever inflation exposure is left.

Suppose instead pension liabilities are fully indexed $\left(D_\pi^l = 0\right)$. In this case the optimal hedge ratio is 100%.

$$\phi = -\frac{D_\pi^n}{D_\pi^{swap}} = -\frac{-10}{10} = 1 \qquad \textbf{14)}$$

The asset liability manager has to completely hedge out the inflation risk (nominal bond prices fall when inflation rises) as pension liabilities in this example do not carry any inflation risk. The portfolio of nominal bonds needs to be protected against rising inflation by an offsetting inflation hedge. Inflation swaps can be used to transform any nominal bond into a real bond plus the respective yield pick up. That is why it has been sometimes recommended to invest into corporate bonds and inflation swaps to make pension assets work

[13] It is sometimes claimed that equities provide an acceptable hedge for real liabilities. Empirically equities show little correlation to changes in (both expected and unexpected) inflation. Fighting risk with another layer of uncorrelated risk increases total risk, rather than reducing it.

[14] An extensive treatment of the calculation of duration vectors for pension liabilities can be found in Scherer (2005).

[15] In our notation $D_\pi^{swap} = \frac{dswap}{d\pi}\frac{1}{swap}$ from (8).

harder. In fact derivatives allow us to decouple interest, inflation and credit risk. This allows pension funds to choose any warranted exposure. Alternatively if liabilities are nominal and hence $D_\pi^l = D_\pi^n$, the optimal inflation hedge becomes zero.

Let us further generalize (13) by introducing basis risk. While the same inflation process drives both nominal bonds as well as the inflation swaps, pension liabilities do not move in tandem. For example pension liabilities might be partially indexed to German banking sector wage growth, π^*, while liability swaps as well as nominal bonds react to EMU CPI ex tobacco, π. Let the relation between both measures be

$$\Delta\pi^* = \theta\Delta\pi + \varepsilon \qquad \mathbf{15)}$$

to allows the decomposition of liability risk into hedgeable inflation risk and non-hedgeable sector specific wage growth. Inflation risk can now be expressed as

$$\begin{aligned}\Pi &= Var\left(D_\pi^l \Delta\pi^* - \left(D_\pi^n + \phi D_\pi^{swap}\right)\Delta\pi\right) \\ &= \left(D_\pi^l\right)^2 \left(\theta^2 Var(\Delta\pi) + Var(\varepsilon)\right) + \left(D_\pi^n + \phi D_\pi^{swap}\right)^2 Var(\Delta\pi) - 2D_\pi^l \left(D_\pi^n + \phi D_\pi^{swap}\right)\theta Var(\Delta\pi)\end{aligned} \qquad \mathbf{16)}$$

Note that $\left(D_\pi^l\right)^2 Var(\varepsilon)$ can not be hedged. It is the part of wage growth that is unrelated to inflation changes. If (15) holds poorly (i.e. the regression exhibits a low R^2 as in the case of equities or commodities as a hedging instrument) this risk may be substantial. Running the risk that assets and liabilities drift apart (surplus risk) in a pension fund essentially uses corporate balance sheet, therefore crowding out real investment projects. Surplus risk in the pension fund requires the plan sponsor to hold equivalent risk capital on her balance sheet in order to absorb periods of bad performance. As risk capital needs to be held in riskless assets, it can not be spent any more in projects with positive net present value.[16] Taking first order derivatives of (16) with respect to ϕ yields

$$\frac{d\Pi}{d\phi} = 2\left(D_\pi^n + \phi D_\pi^{swap}\right)D_\pi^{swap} Var(\Delta\pi) - 2D_\pi^l D_\pi^{swap}\theta Var(\Delta\pi) = 0 \qquad \mathbf{17)}$$

The optimal hedge-ratio becomes then

$$\phi = \frac{\theta D_\pi^l - D_\pi^n}{D_\pi^{swap}} \qquad \mathbf{18)}$$

Again the interpretation is straightforward. If there is no relation at all between the inflation process, driving nominal bonds and inflation swaps 100% inflation hedging would be optimal. We arrive at this result because inflation risk is always on top of (real) interest rate risk.[17] If the risk due to $\Delta\pi^*$ can not be hedged anyway it makes sense to at least reduce the inflation risk in pension assets. It is preferable to use inflation swaps on top of a nominal bond portfolio as this allows a building block approach that uses the most liquid asset categories to build first order interest exposure (government bonds, interest rate swaps) and credit exposure (credit default swaps) rather than using the less liquid inflation linked market (for example TIPS) which is limited in terms of maturity and issuer diversity.

Finally from an accounting point of view it makes sense to enforce $\Delta\pi = \Delta\pi^*$ by projecting future inflation with the same inflation rate inherent in inflation swaps. This ensures that inflation risk in accounting terms can according to (12) be set to zero even though economically there is a difference between banking sector wage inflation and EMU CPI ex tobacco. However be aware that accounting values will become market values as soon as cash payments are being made. Liability valuations might be miss-represented at a time in accounting, but at some stage the economic value has to be paid.

[16] Remember that all diversified capital market investments (so called beta exposure) have a net present value of zero.
[17] Also we made the implicit assumption, that real interest rates (real variable) and inflation (nominal variable) are uncorrelated.

5. Reviewing the Rationale for Inflation Hedging

The previous section has been concerned with the practicalities of inflation hedging, i.e. how can we best hedge inflation risk without interfering with the underlying asset management process. This was an asset management centric question. But should we hedge inflation risk in pension plans in the first place and if to what degree? This is a corporate finance question.

The answer to the question "when should firms hedge?" is simple. A reduction in cash flow volatility will increase firm value, if it increases expected cash flows. While volatility is irrelevant for total firm value in a Modigliani/Miller world, risk reduction carries a positive return, if option-like claims from corporate outsiders are reduced. Examples are the reduction of expected tax payments, expected frictional bankruptcy costs or the protection of costly liquidity.[18] How does a corporate pension fund fit into this picture? The possibility of pension fund deficits places a contingent claim on the plan sponsors assets. In case of pension fund default, the pension fund will demand additional funding from the plan sponsor. At best this results in a drain of costly liquidity while it could also trigger default if the deficit is sufficiently large. The larger the mismatch between pension assets and liabilities, i.e. the more volatile the pension surplus, the more likely this is to happen with the well known adverse effects on current valuation. The natural position for plan sponsors is therefore liability matching. Another way to look at this is to realize that for a publicly traded company, i.e. a company held by a diversified base of investors all beta exposure has zero net present value. There is no trade off between risk and return with the exception of the negative impact of surplus volatility on corporate value.

We have established the case for hedging, but the next question to ask in a risk management context is: "how big is the economic exposure?" Let us recall two key concepts in valuing liabilities from US GAAP accounting. The projected benefit obligation (PBO) is the present value of (currently earned) benefit payments based on projected future salaries. In contrast the accumulated benefit obligation (ABO) does not include further salary increases as they might take place (at the discretion of management) but are not guaranteed yet. Most pension plans target the PBO. However PBO calculations include anticipated but neither granted nor guaranteed future wage increases - that arise from inflation, career trend or productivity gains - as current liabilities. This is troublesome for several reasons. For a start, most inputs a plan sponsor requires like wages, raw materials, energy etc. are inflation sensitive, but no company would account for these as current liability. To make this argument very drastic the direct impact from a future wage increase at Deutsche Bank has a first order importance on its P&L, but is left unaccounted for, while the second order impact on the pension fund, which is really only a derivative of the wage bill is accounted for. Clearly this makes little sense and is not even consistent with a going concern approach. The latter approach would not only require to reflect future wage increases as current liabilities, but also to account for future profits generated by the same employees as current assets. Given the fact that wage increases are only awarded if the economic situation of the company allows so, every pension plan for a profitable company must by definition be over-funded. At the same time plan sponsors face inflation indexation that has already been awarded to pensioners. This can no longer be reversed. Clearly inflation risks from these pension liabilities need to be hedged in order to minimize the pension funds conditional claim if the plan sponsor defaults on the sponsors assets. Otherwise the pension fund consumes corporate balance sheet and crowds out investments into positive NPV projects.

Given the remaining inflation exposure, i.e. recognizing that the plan sponsor issued real bonds to its employees, should this part be hedged? Again the answer is opposite to traditional thinking. Many companies already have inbuilt inflation protection via their revenue stream. To the degree that corporate revenues are inflation sensitive, i.e. show a positive correlation with inflation, there is little reason to hedge inflation risk. Why should a plan sponsor with indexed revenues and indexed pension liabilities use financial contracts to hedge inflation risk that is already hedged via its core business? Essentially this will lead to over-hedging, i.e. creating additional exposure/volatility rather than reducing it. The author concludes that pension liabilities have some economic inflation exposure, but it is likely to be smaller than conventionally thought.

[18] See Doherty N. (2000), Integrated Risk Management, McGraw-Hill for a comprehensive review.

Summary

Inflation swaps offer a convenient building block for liability driven investing. Not only do they offer a flexible instrument to manage differing maturities and inflation definitions, but they also offer easily accessible information about market implied inflation. This information can in turn be used to derive market consistent accounting figures. Any given portfolio can be overlaid with a particular swap structure to create the desired inflation sensitivity of plan assets.

While inflation swaps make inflation hedging highly practical it is not clear that it is equally desirable for corporate pension funds. In fact corporate finance theory dictates us that there is only limited scope for it. Only indexation that has already been guaranteed needs to be hedged. If on top of this, corporate revenues already show strong positive correlation with inflation, the case for hedging is further weakened. Never look at the pension fund in isolation.

Bibliography

Anderson N. and J. Sleath (2001), New Estimates of the UK Real and Nominal Yield Curves, Bank of England Discussion Paper

Benaben E. (2005), Inflation Linked Products, Riskwaters: London

Doherty N. (2000), Integrated Risk Management, McGraw-Hill

Duffie D. and K.J. Singleton (2003), Credit Risk: Pricing, Measurement, and Management, Princeton University Press.

Evans M. (1998), Real Rates, Expected Inflation and Inflation Risk Premia, Journal of Finance, v51, p.187-218

Scherer B. (2005), Liability Hedging and Portfolio Choice, Riskwaters: London

Scherer B. (2006), The Corporate View, Life & Pensions, February, p.34-39

Tuckman B. (1996), Fixed Income Securities, Wiley

Wilkie, A.D. (1986), A stochastic investment model for actuarial use", Transactions of the Faculty of Actuaries, 39, 341-403.

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