# A simplified credit risk model for emerging markets, that measures concentration risk and explicitly relates credit risk to capital adequacy and single obligor limits.

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# Key differences in current credit risk models

- Credit Risk Models:

  Mark to market (Credit Metrics™)

  Default models (CreditRisk™)

- Functional form of probability distributions and relation to loan loss distribution

# Objective of the model

- A theoretical framework that allows ex-ante detection of risk concentration
- Attempt is to measure the concentration of the loan portfolio as it relates to the capital adequacy.

# Assumptions of the model (simple case)

1. Relevant parameters for measuring credit risk are explicitly determined
2. Default probabilities of loans and their covariances are given, default probabilities are homogenous and independent from each other, for all loans along the dimension where loan concentration can occur.

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3. The loss distribution can be characterizes by its mean and variance
4. There is only one possible dimension of loan concentration
5. Nothing is recovered from default.

- Traditional measures by banks to deal with loan concentration-single obligor limits, as a percentage of capital. Lacks informational content about the portfolio distribution.

# Herfindahl-Hirschman Index

- HHI" means the Herfindahl-Hirschman Index, a commonly accepted measure of market concentration.
- It is calculated by squaring the market share of each firm competing in the market and then summing the resulting numbers. For example, for a market consisting of four firms with shares of thirty, thirty, twenty and twenty percent, the HHI is 2600 ($30^2 + 30^2 + 20^2 + 20^2 = 2600$).

# …..continued

- The HHI takes into account the relative size and distribution of the firms in a market and approaches zero when a market consists of a large number of firms of relatively equal size.
- The HHI increases both as the number of firms in the market decreases and as the disparity in size between those firms increases.

# The simplest model

- If all loans have the same default probability "$p$", and assuming independence, one can define "$N$" binary random loss variables "$x_i$" as:

- $$x_i = \begin{cases} f_i & \text{with probability } p \\ 0 & \text{with probability } 1-p \end{cases}$$

Clearly $E(x_i) = pf_i$ and Variance$(x_i) = p(1-p)f_i^2$. Since the variables are independent:

Value of loan portfolio

a) $\mu = E\left(\sum_{i=1}^{N} x_i\right) = \sum_{i=1}^{N} pf_i = pV$ ; where $V = \sum_{i=1}^{N} f_i$

b) $\sigma^2 = Variance\left(\sum_{i=1}^{N} x_i\right) = \sum_{i=1}^{N} Variance(x_i) = p(1-p)\sum_{i=1}^{N} f_i^2$

- Assuming normal distribution for the portfolio loan loss,

$$VAR_{\alpha} = \mu + z_{\alpha}\sigma = pV + z_{\alpha}\sqrt{p(1-p)\sum_{i=1}^{N} f_i^{\,2}}$$

- This is manipulated to yield,

$$H(F) = \frac{\sum_{i=1}^{N} f_i^{\,2}}{\left(\sum_{i=1}^{N} f_i\right)^2} \leq \frac{(\psi - p)^2}{z_{\alpha}^2\, p(1-p)} = \Theta(p, \psi, \alpha)$$

# Capital Adequacy Inequality

- Thus model allows for the following general capital adequacy framework as a measure of loan concentration risk.

Capitalization ratio ← $$\psi \geq p + z_{\alpha}\sqrt{p(1-p)H(\mathbf{F})}$$ → Concentration index

- Thus it is a function of default rates, confidence level used for VAR and the concentration index (HHI).

# Conclusions

- *1.* If the concentration measure exceeds the bound (i.e. $H(F) > \Theta(p,\psi,\alpha)$), then the capital of the bank is at risk, for the given confidence level.

- 2. If the default probability "p" exceeds the capitalization ratio "$\psi$", then the capital of the bank is at risk for any confidence level, regardless of the concentration of the loan portfolio.

- 3. If $\Theta(p,\psi,\alpha) > 1$, no degree of concentration of the loan portfolio, places the capital of the bank at risk.

# Example: Simple loan portfolio

| N° of loans | RATING | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Créditos | A | B | C | D | E | F | G | TOTAL |
| 1 | $4,728 | $5,528 | $3,138 | $5,320 | $1,800 | $1,933 | $358 | $22,805 |
| 2 | $7,728 | $5,848 | $3,204 | $5,765 | $5,042 | $2,317 | $1,090 | $30,994 |
| 3 | | | $4,831 | $20,239 | $15,411 | $2,411 | $2,652 | $45,544 |
| 4 | | | $4,912 | | | $2,598 | $4,929 | $12,439 |
| 5 | | | $5,435 | | | | $6,467 | $11,902 |
| 6 | | | | | | | $6,480 | $6,480 |
| **TOTAL** | $12,456 | $11,376 | $21,520 | $31,324 | $22,253 | $9,259 | $21,976 | **$130,164** |

- Lets assume that this bank has a capital base of $35,000. Based on the loan portfolio info and the default rates and assuming normality at 95% level confidence, is the bank sufficiently capitalized.

| Rating | Default Prob. |
|---|---|
| A | 1.65 |
| B | 3.00 |
| C | 5.00 |
| D | 7.50 |
| E | 10.00 |
| F | 15.00 |
| G | 30.00 |

- Based on the above figures lets assume that the weighted avg default rate for the portfolio is 10.89% and the concentration index(HHI) is 6.61%.

- Therefore

$$\psi \geq p + z_{\alpha}\sqrt{p(1-p)H(F)} = 0.1089 + 1.96\sqrt{0.1089 \times .8911 \times 0.0661} = 0.2658$$

- Therefore the bank’s economic capital must be at least
- $VAR_{1.96}$ = 0.2658 x V (value of loan portfolio) = 0.2658 x $130.164 = **$34,602**
- The bank is therefore adequately capitalized.

- Given the existing parameters, the maximum concentration that the portfolio can bear without jeopardising bank capital is,

$$H(F) = \frac{\sum_{i=1}^{N} f_i^{\,2}}{\left(\sum_{i=1}^{N} f_i\right)^2} \leq \frac{(\psi - p)^2}{z_\alpha^2\, p(1-p)} = \Theta(p, \psi, \alpha)$$

$$\frac{(\psi - p)^2}{z_\alpha^2\, p(1-p)} = \frac{(0.2689 - 0.1089)^2}{1.96^2 \times 0.1089 \times 0.8911} = 0.0687$$

# Accounting for correlation

- Assume that the portfolio loss distribution can be characterized by its mean and its variance and that the vector of default probabilities "π" and the Co-variance matrix "**M**" are given exogenously.
- The VAR to Capital inequality is now

$$VAR_{\alpha} = \pi^{T}\mathbf{F} + z_{\alpha}\sqrt{\mathbf{F}^{\mathrm{T}}\mathbf{M}\,\mathbf{F}} \leq K$$

- The capital adequacy inequality is now,

$$\psi \geq \bar{p} + z_{\alpha} \sqrt{\frac{F^T MF}{F^T F} H(F)} = \bar{p} + z_{\alpha}\, \sigma \sqrt{H(F)}$$

- Where $s^2 = \dfrac{F^T MF}{F^T F}$ ,

- And $\bar{p} = \dfrac{\pi^T \mathbf{F}}{V}$ = wt avg probability of default

- Therefore proceeding further one obtains the limit on loan concentration and single obligor limit as,

$$H(F) \leq \theta \leq \left( \frac{\psi - \bar{p}}{z_{\alpha} \sigma} \right)^{2}$$

- The correlation effect on the concentration index is, assuming equal default probability *p* correlation $\rho$ the covariance between two loans(i,j) is,

$$s_{ij} = s_i s_j r_{ij} = \sqrt{p_i(1-p_i)}\sqrt{p_j(1-p_j)}\, r_{ij} = p(1-p)\, r$$

- And the correlation adjusted concentration index is,

$$H' = \rho + (1 - \rho)H(F)$$

# Example: Simple loan portfolio(adjusted for correlation effect)

- Assuming normality at 95% confidence level,

$$VaR_{.05} = \pi^T F + z_{.05}\sqrt{F^T MF} = 14{,}179 + 1.96(21{,}176) = \$55{,}683$$

- This is 60% higher in the previous uncorrelated example

- We know wt avg default rate is 10.89% and concentration index is 6.61%, therefore, variance of losses is,

$$\sigma = \sqrt{\frac{\mathbf{F}^{\mathrm{T}}\mathbf{M}\mathbf{F}}{\mathbf{F}^{\mathrm{T}}\mathbf{F}}} = \sqrt{0.4006} = 0.6329$$

- Therefore to satisfy the capital adequacy inequality in a correlated portfolio,

$$\psi > \overline{p} + z_{\alpha}\sigma\sqrt{H(F)} = 0.4278$$

- The single obligor limit in this case is,

$$\theta \le \left(\frac{\psi - \overline{p}}{z_{\alpha}\sigma}\right)^2 = \left(\frac{0.35 \quad - 0.1089}{1.96(0.6329)}\right)^2 = 0.0377$$

- That is

$$f_i \le 0.0377 \times \$130{,}164 = \$4917$$

- The impact of correlation is,

$$\rho = \frac{[0.4006 - 0.0978] \times 0.0661}{0.0978 \times [1 - 0.0661]} = 0.2191$$

- And the correlation adjusted concentration index is,
- *H’ = 0.2191 + (1-0.2191)×0.0661 = 0.2707*

- This increases the unexpected loan loss to $0.6329 \times (27.07)^{1/2} = 0.1627$ as against $(p(1-p)H(F))^{1/2} = 0.0801$

- End of session